# SEQUENCES AND SERIES — *Answers*

**1** **a** $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$
$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$

**b** $= 2^5 + 5(2^4)(-x) + 10(2^3)(-x)^2 + 10(2^2)(-x)^3 + 5(2)(-x)^4 + (-x)^5$
$= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$

**c** $= 3^3 + 3(3^2)(10x^2) + 3(3)(10x^2)^2 + (10x^2)^3$
$= 27 + 270x^2 + 900x^4 + 1000x^6$

**d** $= a^5 + 5a^4(2b) + 10a^3(2b)^2 + 10a^2(2b)^3 + 5a(2b)^4 + (2b)^5$
$= a^5 + 10a^4b + 40a^3b^2 + 80a^2b^3 + 80ab^4 + 32b^5$

**e** $= (x^2)^3 + 3(x^2)^2(-y) + 3(x^2)(-y)^2 + (-y)^3$
$= x^6 - 3x^4y + 3x^2y^2 - y^3$

**f** $= 5^4 + 4(5^3)(\tfrac{1}{2}x) + 6(5^2)(\tfrac{1}{2}x)^2 + 4(5)(\tfrac{1}{2}x)^3 + (\tfrac{1}{2}x)^4$
$= 625 + 250x + \tfrac{75}{2}x^2 + \tfrac{5}{2}x^3 + \tfrac{1}{16}x^4$

**g** $= x^4 + 4x^3(\frac{1}{x}) + 6x^2(\frac{1}{x})^2 + 4x(\frac{1}{x})^3 + (\frac{1}{x})^4$
$= x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$

**h** $= t^3 + 3t^2(-\frac{2}{t^2}) + 3t(-\frac{2}{t^2})^2 + (-\frac{2}{t^2})^3$
$= t^3 - 6 + \frac{12}{t^3} - \frac{8}{t^6}$

**2** **a** $= 1 + 6(3x) + \tfrac{6\times5}{2}(3x)^2 + \tfrac{6\times5\times4}{3\times2}(3x)^3 + \ldots$
$= 1 + 18x + 135x^2 + 540x^3 + \ldots$

**b** $= 1 + 8(-\tfrac{1}{4}x) + \tfrac{8\times7}{2}(-\tfrac{1}{4}x)^2 + \tfrac{8\times7\times6}{3\times2}(-\tfrac{1}{4}x)^3 + \ldots$
$= 1 - 2x + \tfrac{7}{4}x^2 - \tfrac{7}{8}x^3 + \ldots$

**c** $= 5^7 + 7(5^6)(-x) + \tfrac{7\times6}{2}(5^5)(-x)^2 + \tfrac{7\times6\times5}{3\times2}(5^4)(-x)^3 + \ldots$
$= 78\,125 - 109\,375x + 65\,625x^2 - 21\,875x^3 + \ldots$

**d** $= 3^{10} + 10(3^9)(2x^2) + \tfrac{10\times9}{2}(3^8)(2x^2)^2 + \tfrac{10\times9\times8}{3\times2}(3^7)(2x^2)^3 + \ldots$
$= 59\,049 + 393\,660x^2 + 1\,180\,980x^4 + 2\,099\,520x^6 + \ldots$

**3** **a** $= \binom{15}{3} = 455$

**b** $= \binom{12}{4} \times (-2)^4 = 7920$

**c** $= \binom{7}{2} \times 3^5 = 5103$

**d** $= \binom{10}{5} \times 2^5 \times (-1)^5 = -8064$

**e** $= \binom{8}{5} \times 2^3 = 448$

**f** $= \binom{9}{3} \times (-1)^3 = -84$

**4** **a** $= (\sqrt{2})^4 + 4(\sqrt{2})^3(-\sqrt{5}) + 6(\sqrt{2})^2(-\sqrt{5})^2 + 4(\sqrt{2})(-\sqrt{5})^3 + (-\sqrt{5})^4$
$= 4 - 8\sqrt{10} + 60 - 20\sqrt{10} + 25$
$= 89 - 28\sqrt{10}$

**b** $= (\sqrt{2})^3 + 3(\sqrt{2})^2(\frac{1}{\sqrt{3}}) + 3(\sqrt{2})(\frac{1}{\sqrt{3}})^2 + (\frac{1}{\sqrt{3}})^3$
$= 2\sqrt{2} + 2\sqrt{3} + \sqrt{2} + \tfrac{1}{9}\sqrt{3}$
$= 3\sqrt{2} + \tfrac{19}{9}\sqrt{3}$

**c** $= 1 + 3(\sqrt{5}) + 3(\sqrt{5})^2 + (\sqrt{5})^3 - [1 + 3(-\sqrt{5}) + 3(-\sqrt{5})^2 + (-\sqrt{5})^3]$
$= 1 + 3\sqrt{5} + 15 + 5\sqrt{5} - [1 - 3\sqrt{5} + 15 - 5\sqrt{5}]$
$= 16 + 8\sqrt{5} - [16 - 8\sqrt{5}]$
$= 16\sqrt{5}$

**5** **a** $= 1 + 10(\frac{x}{2}) + \frac{10\times9}{2}(\frac{x}{2})^2 + \frac{10\times9\times8}{3\times2}(\frac{x}{2})^3 + \ldots$
$= 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \ldots$

**b** **i** let $x = 0.01$
$1.005^{10} \approx 1 + 0.05 + 0.001\,125 + 0.000\,015$
$= 1.051\,14$ (5dp)

**ii** let $x = -0.008$
$0.996^{10} \approx 1 - 0.040 + 0.000\,720 - 0.000\,007\,680$
$= 0.960\,71$ (5dp)

**6** **a** $= 3^8 + 8(3^7)x + \frac{8\times7}{2}(3^6)x^2 + \frac{8\times7\times6}{3\times2}(3^5)x^3 + \ldots$
$= 6561 + 17\,496x + 20\,412x^2 + 13\,608x^3 + \ldots$

**b** **i** let $x = 0.001$
$3.001^8 \approx 6561 + 17.496 + 0.020\,412 + 0.000\,013\,608$
$= 6578.516$ (7sf)

**ii** let $x = -0.005$
$2.995^8 \approx 6561 - 87.480 + 0.510\,300 - 0.001\,701\,000$
$= 6474.029$ (7sf)

**7** **a** $(1 + 10x)^4 = 1 + 4(10x) + 6(10x)^2 + 4(10x)^3 + (10x)^4$
$= 1 + 40x + 600x^2 + 4000x^3 + 10\,000x^4$
$\therefore (1 + 10x)^4 + (1 - 10x)^4$
$= 1 + 40x + 600x^2 + 4000x^3 + 10\,000x^4 + (1 - 40x + 600x^2 - 4000x^3 + 10\,000x^4)$
$= 2 + 1200x^2 + 20\,000x^4$

**b** $(2 + \frac{1}{3}x)^3 = 2^3 + 3(2^2)(\frac{1}{3}x) + 3(2)(\frac{1}{3}x)^2 + (\frac{1}{3}x)^3$
$= 8 + 4x + \frac{2}{3}x^2 + \frac{1}{27}x^3$
$\therefore (2 - \frac{1}{3}x)^3 - (2 + \frac{1}{3}x)^3$
$= 8 - 4x + \frac{2}{3}x^2 - \frac{1}{27}x^3 - (8 + 4x + \frac{2}{3}x^2 + \frac{1}{27}x^3)$
$= -8x - \frac{2}{27}x^3$

**c** $= (1 + 4y)(1 + 3y + 3y^2 + y^3)$
$= 1 + 3y + 3y^2 + y^3 + 4y + 12y^2 + 12y^3 + 4y^4$
$= 1 + 7y + 15y^2 + 13y^3 + 4y^4$

**d** $= (1 - x)(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3})$
$= 1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3} - x - 3 - \frac{3}{x} - \frac{1}{x^2}$
$= -x - 2 + \frac{2}{x^2} + \frac{1}{x^3}$

**8** **a** $= (1 + x^2)[1 + 10(-3x) + \frac{10\times9}{2}(-3x)^2 + \frac{10\times9\times8}{3\times2}(-3x)^3 + \ldots]$

$= (1 + x^2)[1 - 30x + 405x^2 - 3240x^3 + \ldots]$

$= 1 - 30x + 405x^2 - 3240x^3 + x^2 - 30x^3 + \ldots$

$= 1 - 30x + 406x^2 - 3270x^3 + \ldots$

**b** $= (1 - 2x)[1 + 8x + \frac{8\times7}{2}x^2 + \frac{8\times7\times6}{3\times2}x^3 + \ldots]$

$= (1 - 2x)[1 + 8x + 28x^2 + 56x^3 + \ldots]$

$= 1 + 8x + 28x^2 + 56x^3 - 2x - 16x^2 - 56x^3 + \ldots$

$= 1 + 6x + 12x^2 + \ldots$

**c** $= (1 + x + x^2)[1 + 6(-x) + \frac{6\times5}{2}(-x)^2 + \frac{6\times5\times4}{3\times2}(-x)^3 + \ldots]$

$= (1 + x + x^2)[1 - 6x + 15x^2 - 20x^3 + \ldots]$

$= 1 - 6x + 15x^2 - 20x^3 + x - 6x^2 + 15x^3 + x^2 - 6x^3 + \ldots$

$= 1 - 5x + 10x^2 - 11x^3 + \ldots$

**d** $= (1 + 3x - x^2)[1 + 7(2x) + \frac{7\times6}{2}(2x)^2 + \frac{7\times6\times5}{3\times2}(2x)^3 + \ldots]$

$= (1 + 3x - x^2)[1 + 14x + 84x^2 + 280x^3 + \ldots]$

$= 1 + 14x + 84x^2 + 280x^3 + 3x + 42x^2 + 252x^3 - x^2 - 14x^3 + \ldots$

$= 1 + 17x + 125x^2 + 518x^3 + \ldots$

**9** **a** $= \binom{8}{4} \times y^4 \times (\frac{1}{y})^4 = 70$

**b** $= \binom{12}{6} \times (2y)^6 \times (-\frac{1}{2y})^6 = 924$

**c** $= \binom{6}{2} \times (\frac{1}{y})^4 \times (y^2)^2 = 15$

**d** $= \binom{9}{3} \times (3y)^6 \times (-\frac{1}{y^2})^3 = -61\,236$

**10** **a** $\frac{n(n-1)}{2} \times (\frac{2}{5})^2 = 1.6$

$n(n - 1) = \frac{25}{2} \times 1.6 = 20$

$n^2 - n - 20 = 0$

$(n + 4)(n - 5) = 0$

$n > 0 \;\; \therefore\; n = 5$

**b** $= 5 \times (\frac{2}{5})^4 = \frac{16}{125}$ or 0.128

**11** **a** $y_1 = (1 - 2x)[1 + 10x + \frac{10\times9}{2}x^2 + \ldots]$

$= 1 + 10x + 45x^2 - 2x - 20x^2 + \ldots$

$= 1 + 8x + 25x^2 + \ldots$

$\therefore\; a = 25,\ b = 8,\ c = 1$

**b** $x = 0.2$: $y_1 = 0.6 \times (1.2)^{10} = 3.71504$

$y_2 = (25\times0.04) + (8\times0.2) + 1 = 3.6$

% error $= \frac{3.71504 - 3.6}{3.71504} \times 100\% = 3.1\%$ (2sf)

**12** **a** $(1 + px)^q = 1 + q(px) + \frac{q(q-1)}{2}(px)^2 + \ldots$

$\therefore\; pq = -12$ and $\frac{1}{2}p^2q(q - 1) = 60$

sub. $p = -\frac{12}{q}$

$\Rightarrow \frac{72}{q}(q - 1) = 60$

$72(q - 1) = 60q$

$q = 6,\ p = -2$

**b** $= \frac{6\times5\times4}{3\times2} \times (-2)^3 = -160$

**13** **a** $= 3^{12} + 12(3^{11})(-\frac{x}{3}) + \frac{12\times11}{2}(3^{10})(-\frac{x}{3})^2 + \frac{12\times11\times10}{3\times2}(3^9)(-\frac{x}{3})^3 + \ldots$

$= 531\,441 - 708\,588x + 433\,026x^2 - 160\,380x^3 + \ldots$

**b** let $\frac{x}{3} = 0.002 \quad \therefore x = 0.006$

$2.998^{12} \approx 531\,441 - 4251.528 + 15.588\,936 - 0.034\,642\,080$

$= 527\,205.03$ (2dp)

**14** **a** $= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$

**b** $= 3 - 2\sqrt{3} + \sqrt{3} - 2 = 1 - \sqrt{3}$

**c** **i** $= [(\sqrt{3} + 1)(\sqrt{3} - 2)]^5 = (1 - \sqrt{3})^5$

$= 1 - 5(\sqrt{3}) + 10(\sqrt{3})^2 - 10(\sqrt{3})^3 + 5(\sqrt{3})^4 - (\sqrt{3})^5$

$= 1 - 5\sqrt{3} + 30 - 30\sqrt{3} + 45 - 9\sqrt{3}$

$= 76 - 44\sqrt{3}$

**ii** $= (\sqrt{3} + 1)(76 - 44\sqrt{3})$

$= 76\sqrt{3} - 132 + 76 - 44\sqrt{3}$

$= -56 + 32\sqrt{3}$

**15** **a** $= 1 + 9(\frac{x}{2}) + \frac{9\times8}{2}(\frac{x}{2})^2 + \frac{9\times8\times7}{3\times2}(\frac{x}{2})^3 + \frac{9\times8\times7\times6}{4\times3\times2}(\frac{x}{2})^4 + \ldots$

$= 1 + \frac{9}{2}x + 9x^2 + \frac{21}{2}x^3 + \frac{63}{8}x^4 + \ldots$

**b** $= \frac{21}{2} - (-\frac{21}{2}) = 21$

**c** $= (1 \times \frac{63}{8}) + (2 \times \frac{21}{2}) = 28\frac{7}{8}$

**16** $10(x^3)^2(\frac{a}{x^2})^3 = -80$

$a^3 = -8$

$a = -2$

**17** **a** $(1 + \frac{x}{k})^n = 1 + n(\frac{x}{k}) + \frac{n(n-1)}{2}(\frac{x}{k})^2 + \frac{n(n-1)(n-2)}{3\times2}(\frac{x}{k})^3 + \ldots$

$\therefore \frac{n(n-1)}{2k^2} = 3 \times \frac{n(n-1)(n-2)}{6k^3}$

$kn(n-1) = n(n-1)(n-2)$

$n(n-1)[k - (n-2)] = 0$

$n > 1 \quad \therefore k - (n-2) = 0$

$k = n - 2$

**b** $k = 7 - 2 = 5$

$(1 + \frac{x}{5})^7 = 1 + 7(\frac{x}{5}) + \frac{7\times6}{2}(\frac{x}{5})^2 + \frac{7\times6\times5}{3\times2}(\frac{x}{5})^3 + \frac{7\times6\times5\times4}{4\times3\times2}(\frac{x}{5})^4 + \ldots$

$= 1 + \frac{7}{5}x + \frac{21}{25}x^2 + \frac{7}{25}x^3 + \frac{7}{125}x^4 + \ldots$